1. Linear Equations
2. Solving Linear Equations and Inequalities: Solving Equations
3. Equations and inequalities: Solving linear equations
4. Equations and inequalities: Linear simultaneous equations

Linear Equations
This chapter covers principles of linear equations. After completing this chapter students should be able to: graph a linear equation; find the slope of a line; determine an equation of a line; solve linear systems; and complete application problems using linear equations.

Chapter Overview

In this chapter, you will learn to:

1. Graph a linear equation.
2. Find the slope of a line.
3. Determine an equation of a line.
4. Solve linear systems.
5. Do application problems using linear equations.

Graphing a Linear Equation

Equations whose graphs are straight lines are called **linear equations**. The following are some examples of linear equations:

$2\mathrm{x} - 3\mathrm{y} = 6$, $3\mathrm{x} = 4\mathrm{y} - 7$, $y = 2\mathrm{x} - 5$, $2\mathrm{y} = 3$, and $x - 2 = 0$.

A line is completely determined by two points, therefore, to graph a linear equation, we need to find the coordinates of two points. This can be accomplished by choosing an arbitrary value for x or y and then solving for the other variable.

Example:
Exercise:

Problem: Graph the line: $y = 3\mathrm{x} + 2$

Solution:

We need to find the coordinates of at least two points.

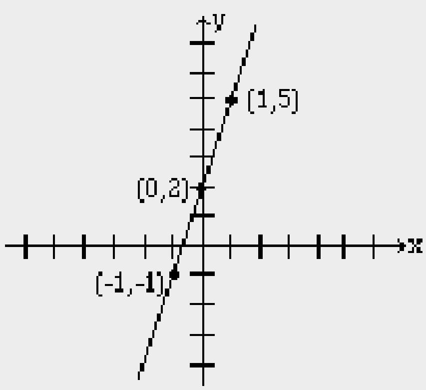
We arbitrarily choose $x = -1$, $x = 0$, and $x = 1$.

If $x = -1$, then $y = 3(-1) + 2$ or -1. Therefore, (–1, –1) is a point on this line.

If $x = 0$, then $y = 3(0) + 2$ or $y = 2$. Hence the point (0, 2).

If $x = 1$, then $y = 5$, and we get the point (1, 5). Below, the results are summarized, and the line is graphed.

X	-1	0	1
Y	-1	2	5



Example:
Exercise:

Problem: Graph the line: 2x $+ y = 4$

Solution:

Again, we need to find coordinates of at least two points.

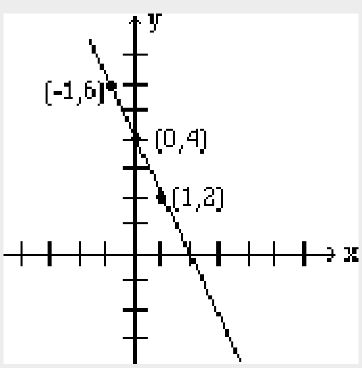
We arbitrarily choose $x = -1$, $x = 0$ and $y = 2$.

If $x = -1$, then $2(-1) + y = 4$ which results in $y = 6$. Therefore, (–1, 6) is a point on this line.

If $x = 0$, then $2(0) + y = 4$, which results in $y = 4$. Hence the point (0, 4).

If $y = 2$, then 2x $+ 2 = 4$, which yields $x = 1$, and gives the point (1, 2). The table below shows the points, and the line is graphed.

x	-1	0	1
y	6	4	2



The points at which a line crosses the coordinate axes are called the **intercepts**. When graphing a line, intercepts are preferred because they are easy to find. In order to find the x-intercept, we let $y = 0$, and to find the y-intercept, we let $x = 0$.

Example:
Exercise:

Problem: Find the intercepts of the line: $2\text{x} - 3\text{y} = 6$, and graph.

Solution:

To find the x-intercept, we let $y = 0$ in our equation, and solve for x.
Equation:

$$2\text{x} - 3(0) = 6$$

Equation:

$$2\text{x} - 0 = 6$$

Equation:

$$2\text{x} = 6$$

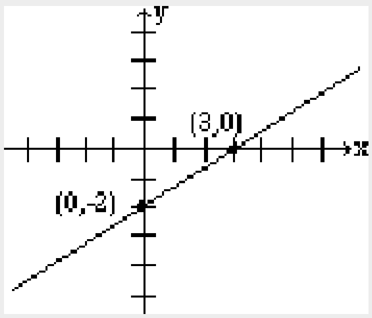
Equation:

$$x = 3$$

Therefore, the x-intercept is 3.

Similarly by letting $x = 0$, we obtain the y-intercept which is -2.

Note: If the x-intercept is 3, and the y-intercept is –2, then the corresponding points are (3, 0) and (0, –2), respectively.



In higher math, equations of lines are sometimes written in parametric form. For example, $x = 3 + 2\text{t}$, $y = 1 + t$. The letter t is called the parameter or the dummy variable. Parametric lines can be graphed by finding values for x and y by substituting numerical values for t.

Example:
Exercise:

Problem:

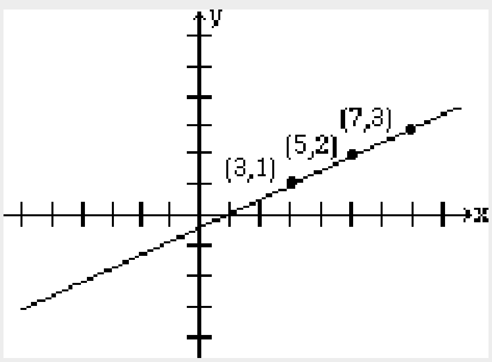
Graph the line given by the parametric equations: $x = 3 + 2\text{t}$, $y = 1 + t$

Solution:

Let $t = 0$, 1 and 2, and then for each value of t find the corresponding values for x and y.

The results are given in the table below.

t	0	1	2
x	3	5	7
y	1	2	3



Horizontal and Vertical Lines

When an equation of a line has only one variable, the resulting graph is a horizontal or a vertical line.

The graph of the line $x = a$, where a is a constant, is a vertical line that passes through the point (a, 0). Every point on this line has the x-coordinate a, regardless of the y-coordinate.

The graph of the line $y = b$, where b is a constant, is a horizontal line that passes through the point (0, b). Every point on this line has the y-coordinate b, regardless of the x-coordinate.

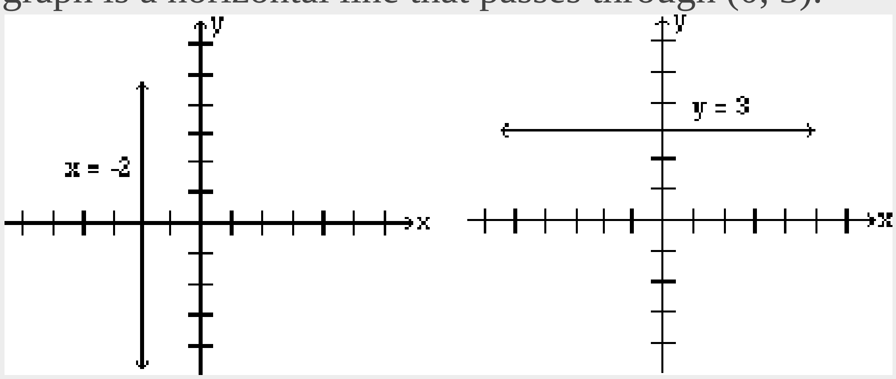
Example:
Exercise:

Problem: Graph the lines: $x = -2$, and $y = 3$.

Solution:

The graph of the line $x = -2$ is a vertical line that has the x-coordinate –2 no matter what the y-coordinate is. Therefore, the graph is a vertical line passing through (–2, 0).

The graph of the line $y = 3$, is a horizontal line that has the y-coordinate 3 regardless of what the x-coordinate is. Therefore, the graph is a horizontal line that passes through (0, 3).



Note: Most students feel that the coordinates of points must always be integers. This is not true, and in real life situations, not always possible. Do not be intimidated if your points include numbers that are fractions or decimals.

Slope of a Line

Section Overview

In this section, you will learn to:

1. Find the slope of a line if two points are given.
2. Graph the line if a point and the slope are given.
3. Find the slope of the line that is written in the form $y = \mathrm{mx} + b$.
4. Find the slope of the line that is written in the form $\mathrm{Ax} + \mathrm{By} = c$.

In the last section, we learned to graph a line by choosing two points on the line. A graph of a line can also be determined if one point and the "steepness" of the line is known. The precise number that refers to the steepness or inclination of a line is called the **slope** of the line.

From previous math courses, many of you remember slope as the "rise over run," or "the vertical change over the horizontal change" and have often seen it expressed as:
Equation:

$$\frac{\text{rise}}{\text{run}}, \frac{\text{vertical change}}{\text{horizontal change}}, \frac{\Delta y}{\Delta x}\text{etc.}$$

We give a precise definition.

If (x_1, y_1) and (x_2, y_2) are two different points on a line, then the slope of the line is

Slope $= m = \frac{y_2 - y_1}{x_2 - x_1}$

Example:
Exercise:

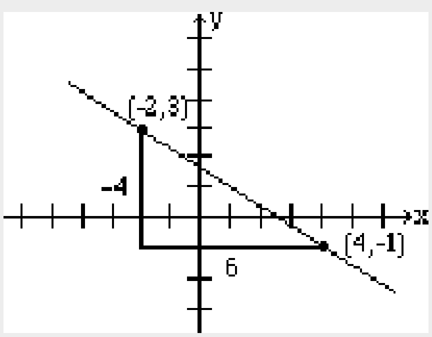
Problem:

Find the slope of the line that passes through the points (-2, 3) and (4, -1), and graph the line.

Solution:

Let $(x_1,y_1) = (-2,3)$ and $(x_2,y_2) = (4, -1)$ then the slope
Equation:

$$m = \frac{-1 - 3}{4 - (-2)} = -\frac{4}{6} = -\frac{2}{3}$$



To give the reader a better understanding, both the vertical change, –4, and the horizontal change, 6, are shown in the above figure.

When two points are given, it does not matter which point is denoted as (x_1,y_1) and which (x_2,y_2). The value for the slope will be the

same. For example, if we choose $(x_1,y_2) = (4,-1)$ and $(x_2,y_2) = (-2,3)$, we will get the same value for the slope as we obtained earlier. The steps involved are as follows.

Equation:

$$m = \frac{3-(-1)}{-2-4} = \frac{4}{-6} = -\frac{2}{3}$$

The student should further observe that if a line rises when going from left to right, then it has a positive slope; and if it falls going from left to right, it has a negative slope.

Example:

Exercise:

Problem:

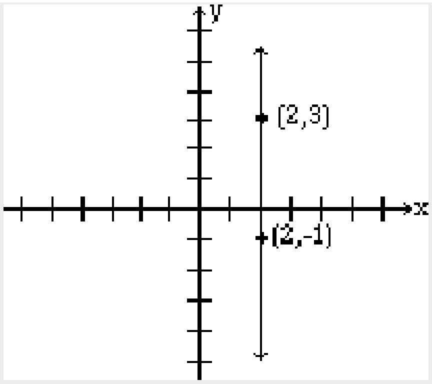
Find the slope of the line that passes through the points (2, 3) and (2, -1), and graph.

Solution:

Let $(x_1,y_1) = (2,3)$ and $(x_2,y_2) = (2,-1)$ then the slope

Equation:

$$m = \frac{-1-3}{2-2} = -\frac{4}{0} = \text{undefined}$$



Note: The slope of a vertical line is undefined.

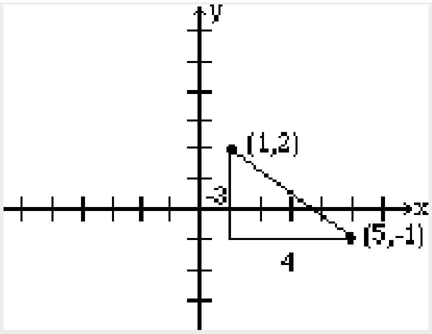
Example:
Exercise:

Problem:

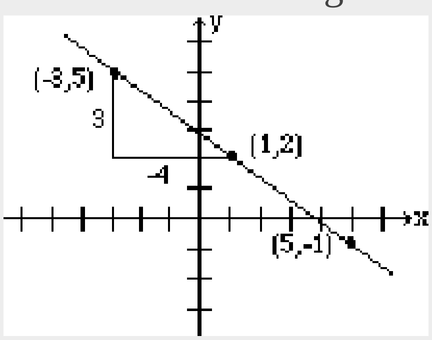
Graph the line that passes through the point (1, 2) and has slope $-\frac{3}{4}$.

Solution:

Slope equals $\frac{\text{rise}}{\text{run}}$. The fact that the slope is $\frac{-3}{4}$, means that for every rise of –3 units (fall of 3 units) there is a run of 4. So if from the given point (1, 2) we go down 3 units and go right 4 units, we reach the point (5, –1). The following graph is obtained by connecting these two points.



Alternatively, since $\frac{3}{-4}$ represents the same number, the line can be drawn by starting at the point (1,2) and choosing a rise of 3 units followed by a run of –4 units. So from the point (1, 2), we go up 3 units, and to the left 4, thus reaching the point (–3, 5) which is also on the same line. See figure below.



Example:

Exercise:

Problem: Find the slope of the line $2x + 3y = 6$.

Solution:

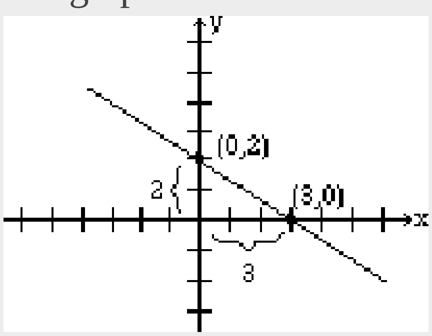
In order to find the slope of this line, we will choose any two points on this line.

Again, the selection of x and y intercepts seems to be a good choice. The x-intercept is (3, 0), and the y-intercept is (0, 2). Therefore, the slope is

Equation:

$$m = \frac{2-0}{0-3} = -\frac{2}{3}.$$

The graph below shows the line and the intercepts: x and y.



Example:
Exercise:

Problem: Find the slope of the line $y = 3\text{x} + 2$.

Solution:

We again find two points on the line. Say (0, 2) and (1, 5).

Therefore, the slope is $m = \frac{5-2}{1-0} = \frac{3}{1} = 3$.

Look at the slopes and the y-intercepts of the following lines.

The line	Slope	y-intercept
$y = 3\text{x} + 2$	3	2
$y = -2\text{x} + 5$	-2	5
$y = 3/2\text{x} - 4$	3/2	-4

It is no coincidence that when an equation of the line is solved for y, the coefficient of the x term represents the slope, and the constant term represents the y-intercept.

In other words, for the line $y = \text{mx} + b$, m is the slope, and b is the y-intercept.

Example:
Exercise:

Problem:

Determine the slope and y-intercept of the line $2x + 3y = 6$.

Solution:

We solve for y.
Equation:

$$2x + 3y = 6$$

Equation:

$$3y = -2x + 6$$

Equation:

$$y = -2/3x + 2$$

The slope $=$ the coefficient of the x term $= -2/3$

The y-intercept $=$ the constant term $= 2$.

Determining the Equation of a Line

Section Overview

In this section, you will learn to:

1. Find an equation of a line if a point and the slope are given.
2. Find an equation of a line if two points are given.

So far, we were given an equation of a line and were asked to give information about it. For example, we were asked to find points on it, find its slope and even find intercepts. Now we are going to reverse the process.

That is, we will be given either two points, or a point and the slope of a line, and we will be asked to find its equation.

An equation of a line can be written in two forms, the **slope-intercept form** or the **standard form**.

The Slope-Intercept Form of a Line: $y = \mathrm{mx} + b$

A line is completely determined by two points, or a point and slope. So it makes sense to ask to find the equation of a line if one of these two situations is given.

Example:
Exercise:

Problem:

Find an equation of a line whose slope is 5, and y-intercept is 3.

Solution:

In the last section we learned that the equation of a line whose slope = m and y-intercept = b is $y = \mathrm{mx} + b$.

Since $m = 5$, and $b = 3$, the equation is $y = 5\mathrm{x} + 3$.

Example:
Exercise:

Problem:

Find the equation of the line that passes through the point (2, 7) and has slope 3.

Solution:

Since $m = 3$, the partial equation is $y = 3\mathrm{x} + b$.

Now b can be determined by substituting the point (2, 7) in the equation $y = 3\mathrm{x} + b$.
Equation:

$$7 = 3(2) + b$$

Equation:

$$b = 1$$

Therefore, the equation is $y = 3\mathrm{x} + 1$.

Example:
Exercise:

Problem:

Find an equation of the line that passes through the points (–1, 2), and (1, 8).

Solution:
Equation:

$$m = \frac{8 - 2}{1 - (-1)} = \frac{6}{2} = 3$$

So the partial equation is $y = 3\mathrm{x} + b$

Now we can use either of the two points (–1, 2) or (1, 8), to determine b.

Substituting (–1, 2) gives
Equation:

$$2 = 3(-1) + b$$

Equation:

$$5 = b$$

So the equation is $y = 3\text{x} + 5$.

Example:
Exercise:

Problem:

Find an equation of the line that has x-intercept 3, and y-intercept 4.

Solution:

x-intercept = 3, and y-intercept = 4 correspond to the points (3, 0), and (0, 4), respectively.
Equation:

$$m = \frac{4-0}{0-3} = \frac{4}{-3}$$

So the partial equation for the line is $y = -4/3\text{x} + b$

Substituting (0, 4) gives
Equation:

$$4 = -4/3(0) + b$$

Equation:

$$4 = b$$

Therefore, the equation is $y = -4/3\text{x} + 4$.

The Standard form of a Line: $\text{Ax} + \text{By} = C$

Another useful form of the equation of a line is the Standard form.

Let L be a line with slope m, and containing a point (x_1, y_1). If (x, y) is any other point on the line L, then by the definition of a slope, we get
Equation:

$$m = \frac{y - y_1}{x - x_1}$$

Equation:

$$y - y_1 = m(x - x_1)$$

The last result is referred to as the **point-slope form** or point-slope formula. If we simplify this formula, we get the equation of the line in the standard form, $\text{Ax} + \text{By} = C$.

Example:
Exercise:

Problem:

Using the point-slope formula, find the standard form of an equation of the line that passes through the point (2, 3) and has slope –3/5.

Solution:

Substituting the point (2, 3) and $m = -3/5$ in the point-slope formula, we get
Equation:

$$y - 3 = -3/5(x - 2)$$

Multiplying both sides by 5 gives us
Equation:

$$5(y - 3) = -3/5(x - 2)$$

Equation:

$$5\text{y} - 15 = -3\text{x} + 6$$

Equation:

$$3\text{x} + 5\text{y} = 21$$

Example:
Exercise:

Problem:

Find the standard form of the line that passes through the points (1, -2), and (4, 0).

Solution:
Equation:

$$m = \frac{0 - (-2)}{4 - 1} = \frac{2}{3}$$

The point-slope form is
Equation:

$$y - (-2) = 2/3(x - 1)$$

Multiplying both sides by 3 gives us
Equation:

$$3(y+2)=2(x-1)$$

Equation:

$$3\text{y}+6=2\text{x}-2$$

Equation:

$$-2\text{x}+3\text{y}=-8$$

Equation:

$$2\text{x}-3\text{y}=8$$

We should always be able to convert from one form of an equation to another. That is, if we are given a line in the slope-intercept form, we should be able to express it in the standard form, and vice versa.

Example:
Exercise:

Problem: Write the equation $y=-2/3\text{x}+3$ in the standard form.

Solution:

Multiplying both sides of the equation by 3, we get
Equation:

$$3\text{y}=-2\text{x}+9$$

Equation:

$$2\text{x}+3\text{y}=9$$

Example:
Exercise:

Problem:

Write the equation $3x - 4y = 10$ in the slope-intercept form.

Solution:

Solving for y, we get

Equation:

$$-4y = -3x + 10$$

Equation:

$$y = 3/4x - 5/2$$

Finally, we learn a very quick and easy way to write an equation of a line in the standard form. But first we must learn to find the slope of a line in the standard form by inspection.

By solving for y, it can easily be shown that the slope of the line $Ax + By = C$ is $-A/B$. The reader should verify.

Example:
Exercise:

Problem: Find the slope of the following lines, by inspection.

a. $3x - 5y = 10$
b. $2x + 7y = 20$
c. $4x - 3y = 8$

Solution:

a. $A = 3$, $B = -5$, therefore, $m = -\frac{3}{-5} = \frac{3}{5}$
b. $A = 2$, $B = 7$, therefore, $m = -\frac{2}{7}$
c. $m = -\frac{4}{-3} = \frac{4}{3}$

Now that we know how to find the slope of a line in the standard form by inspection, our job in finding the equation of a line is going to be very easy.

Example:
Exercise:

Problem:

Find an equation of the line that passes through (2, 3) and has slope $-4/5$.

Solution:

Since the slope of the line is $-4/5$, we know that the left side of the equation is $4x + 5y$, and the partial equation is going to be
Equation:

$$4x + 5y = c$$

Of course, c can easily be found by substituting for x and y.
Equation:

$$4(2) + 5(3) = c$$

Equation:

$$23 = c$$

The desired equation is
Equation:

$$4\text{x} + 5\text{y} = 23.$$

If you use this method often enough, you can do these problems very quickly.

Applications

Now that we have learned to determine equations of lines, we get to apply these ideas in real-life equations.

Example:
Exercise:

Problem:

A taxi service charges $0.50 per mile plus a $5 flat fee. What will be the cost of traveling 20 miles? What will be cost of traveling x miles?

Solution:
Equation:

$$\text{The cost of traveling 20 miles} = y = (.50)(20) + 5 = 10 + 5 = 15$$

Equation:

$$\text{The cost of traveling } x \text{ miles} = y = (.50)(x) + 5 = .50x + 5$$

In this problem, \$0.50 per mile is referred to as the **variable cost**, and the flat charge \$5 as the **fixed cost**. Now if we look at our cost equation $y = .50x + 5$, we can see that the variable cost corresponds to the slope and the fixed cost to the y-intercept.

Example:
Exercise:

Problem:

The variable cost to manufacture a product is \$10 and the fixed cost \$2500. If x represents the number of items manufactured and y the total cost, write the cost function.

Solution:

The fact that the variable cost represents the slope and the fixed cost represents the y-intercept, makes $m = 10$ and $y = 2500$.

Therefore, the cost equation is $y = 10x + 2500$.

Example:
Exercise:

Problem:

It costs \$750 to manufacture 25 items, and \$1000 to manufacture 50 items. Assuming a linear relationship holds, find the cost equation, and use this function to predict the cost of 100 items.

Solution:

We let $x =$ the number of items manufactured, and let $y =$ the cost.

Solving this problem is equivalent to finding an equation of a line that passes through the points (25, 750) and (50, 1000).

$m = \frac{1000-750}{50-25} = 10$

Therefore, the partial equation is $y = 10x + b$

By substituting one of the points in the equation, we get $b = 500$

Therefore, the cost equation is $y = 10x + 500$

Now to find the cost of 100 items, we substitute $x = 100$ in the equation $y = 10x + 500$

So the cost $= y = 10(100) + 500 = 1500$

Example:
Exercise:

Problem:

The freezing temperature of water in Celsius is 0 degrees and in Fahrenheit 32 degrees. And the boiling temperatures of water in Celsius, and Fahrenheit are 100 degrees, and 212 degrees, respectively. Write a conversion equation from Celsius to Fahrenheit and use this equation to convert 30 degrees Celsius into Fahrenheit.

Solution:

Let us look at what is given.

Centigrade	Fahrenheit
0	32
100	212

Again, solving this problem is equivalent to finding an equation of a line that passes through the points (0, 32) and (100, 212).

Since we are finding a linear relationship, we are looking for an equation $y = \text{mx} + b$, or in this case $F = \text{mC} + b$, where x or C represent the temperature in Celsius, and y or F the temperature in Fahrenheit.

Equation:

$$\text{slope } m = \frac{212 - 32}{100 - 0} = \frac{9}{5}$$

The equation is $F = \frac{9}{5}C + b$

Substituting the point (0, 32), we get

Equation:

$$F = \frac{9}{5}C + 32.$$

Now to convert 30 degrees Celsius into Fahrenheit, we substitute $C = 30$ in the equation

Equation:

$$F = \frac{9}{5}C + 32$$

Equation:

$$F = \frac{9}{5}(30) + 32 = 86$$

Example:
Exercise:

Problem:

The population of Canada in the year 1970 was 18 million, and in 1986 it was 26 million. Assuming the population growth is linear, and x represents the year and y the population, write the function that gives a relationship between the time and the population. Use this equation to predict the population of Canada in 2010.

Solution:

The problem can be made easier by using 1970 as the base year, that is, we choose the year 1970 as the year zero. This will mean that the year 1986 will correspond to year 16, and the year 2010 as the year 40.

Now we look at the information we have.

Solving this problem is equivalent to finding an equation of a line that passes through the points (0, 18) and (16, 26).
Equation:

$$m = \frac{26 - 18}{16 - 0} = \frac{1}{2}$$

The equation is $y = \frac{1}{2}x + b$

Substituting the point (0, 18), we get
Equation:

$$y = \frac{1}{2}x + 18$$

Now to find the population in the year 2010, we let $x = 40$ in the equation

Equation:

$$y = \frac{1}{2}x + 18$$

Equation:

$$y = \frac{1}{2}(40) + 18 = 38$$

So the population of Canada in the year 2010 will be 38 million.

Year	Population
0 (1970)	18 million
16 (1986)	26 million

More Applications

Section Overview

In this section, you will learn to:

1. Solve a linear system in two variables.
2. Find the equilibrium point when a demand and a supply equation are given.
3. Find the break-even point when the revenue and the cost functions are given.

In this section, we will do application problems that involve the intersection of lines. Therefore, before we proceed any further, we will first learn how to find the intersection of two lines.

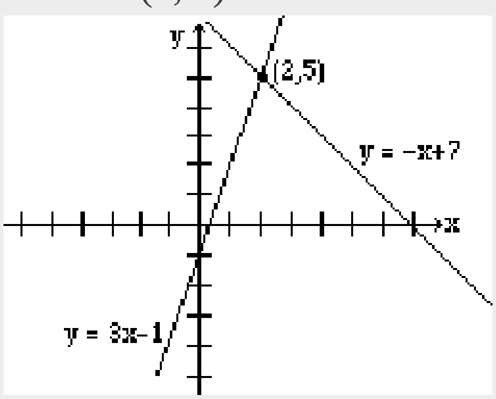
Example:
Exercise:

Problem:

Find the intersection of the line $y = 3\text{x} - 1$ and the line $y = -x + 7$.

Solution:

We graph both lines on the same axes, as shown below, and read the solution (2, 5).



Finding an intersection of two lines graphically is not always easy or practical; therefore, we will now learn to solve these problems algebraically.

At the point where two lines intersect, the x and y values for both lines are the same. So in order to find the intersection, we either let the x-values or the y-values equal.

If we were to solve the above example algebraically, it will be easier to let the y-values equal. Since $y = 3\text{x} - 1$ for the first line, and $y = -x + 7$ for the second line, by letting the y-values equal, we get

Equation:

$$\begin{aligned} &3x - 1 = -x + 7 \\ &4x = 8 \\ &x = 2 \end{aligned}$$

By substituting $x = 2$ in any of the two equations, we obtain $y = 5$.

Hence, the solution (2, 5).

One common algebraic method used in solving systems of equations is called the **elimination method**. The object of this method is to eliminate one of the two variables by adding the left and right sides of the equations together. Once one variable is eliminated, we get an equation that has only one variable for which it can be solved. Finally, by substituting the value of the variable that has been found in one of the original equations, we get the value of the other variable. The method is demonstrated in the example below.

Example:
Exercise:

Problem:

Find the intersection of the lines $2\text{x} + y = 7$ and $3\text{x} - y = 3$ by the elimination method.

Solution:

We add the left and right sides of the two equations.
Equation:

$$\begin{array}{c} 2\text{x} + y = 7 \\ 3\text{x} - y = 3 \\ \hline 5\text{x} = 10 \end{array}$$

Equation:

$$x = 2$$

Now we substitute $x = 2$ in any of the two equations and solve for y.
Equation:

$$2(2) + y = 7$$

Equation:

$$y = 3$$

Therefore, the solution is (2, 3).

Example:
Exercise:

Problem:

Solve the system of equations $x + 2\text{y} = 3$ and $2\text{x} + 3\text{y} = 4$ by the elimination method.

Solution:

If we add the two equations, none of the variables are eliminated. But the variable x can be eliminated by multiplying the first equation by –2, and leaving the second equation unchanged.
Equation:

$$-2\text{x} - 4\text{y} = -6$$
$$2\text{x} + 3\text{y} = 4$$
$$-y = -2$$

Equation:

$$y = 2$$

Substituting $y = 2$ in $x + 2\text{y} = 3$, we get
Equation:

$$x + 2(2) = 3$$

Equation:

$$x = -1$$

Therefore, the solution is (–1, 2).

Example:
Exercise:

Problem:

Solve the system of equations $3x - 4y = 5$ and $4x - 5y = 6$.

Solution:

This time, we multiply the first equation by – 4 and the second by 3 before adding. (The choice of numbers is not unique.)
Equation:

$$-12x + 16y = -20$$
$$12x - 15y = 18$$
$$y = -2$$

By substituting $y = -2$ in any one of the equations, we get $x = -1$. Hence the solution (–1, –2).

Supply, Demand and the Equilibrium Market Price
In a free market economy the supply curve for a commodity is the number of items of a product that can be made available at different prices, and the demand curve is the number of items the consumer will buy at different prices. As the price of a product increases, its demand decreases and supply increases. On the other hand, as the price decreases the demand increases and supply decreases. The **equilibrium price** is reached when the demand equals the supply.

Example:
Exercise:

Problem:

The supply curve for a product is $y = 1.5\text{x} + 10$ and the demand curve for the same product is $y = -2.5\text{x} + 34$, where x is the price and y the number of items produced. Find the following.

a. How many items will be supplied at a price of $10?
b. How many items will be demanded at a price of $10?
c. Determine the equilibrium price.
d. How many items will be produced at the equilibrium price?

Solution:

a. We substitute $x = 10$ in the supply equation, $y = 1.5\text{x} + 10$, and the answer is $y = 25$.
b. We substitute $x = 10$ in the demand equation, $y = -2.5\text{x} + 34$, and the answer is $y = 9$.

c. By letting the supply equal the demand, we get
Equation:

$$1.5\text{x} + 10 = -2.5\text{x} + 34$$

Equation:

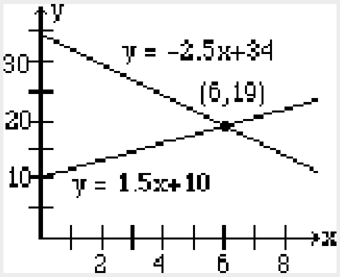
$$4\text{x} = 24$$

Equation:

$$x = 6$$

d. We substitute $x = 6$ in either the supply or the demand equation and we get $y = 19$.

The graph below shows the intersection of the supply and the demand functions and their point of intersection, (6, 19).



Break-Even Point
In a business, the profit is generated by selling products. If a company sells x number of items at a price P, then the revenue R is P times x , i.e., $R = P \cdot x$. The production costs are the sum of the variable costs and the fixed costs, and are often written as $C = \mathrm{mx} + b$, where x is the number of items manufactured.

A company makes a profit if the revenue is greater than the cost, and it shows a loss if the cost is greater than the revenue. The point on the graph where the revenue equals the cost is called the **Break-even point**.

Example:
Exercise:

Problem:

If the revenue function of a product is $R = 5\mathrm{x}$ and the cost function is $y = 3\mathrm{x} + 12$, find the following.

a. If 4 items are produced, what will the revenue be?
b. What is the cost of producing 4 items?
c. How many items should be produced to break-even?
d. What will be the revenue and the cost at the break-even point?

Solution:

a. We substitute $x = 4$ in the revenue equation $R = 5$x, and the answer is $R = 20$.
b. We substitute $x = 4$ in the cost equation $C = 3$x $+ 12$, and the answer is $C = 24$.

c. By letting the revenue equal the cost, we get
Equation:

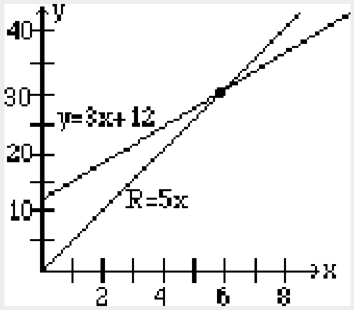
$$5x = 3x + 12$$

Equation:

$$x = 6$$

d. We substitute $x = 6$ in either the revenue or the cost equation, and we get $R = C = 30$.

The graph below shows the intersection of the revenue and the cost functions and their point of intersection, (6, 30).



y
40
30
20
10
y=3x+12
R=5x
2
4
6
8
x

Solving Linear Equations and Inequalities: Solving Equations
This module is from Elementary Algebra by Denny Burzynski and Wade Ellis, Jr. In this chapter, the emphasis is on the mechanics of equation solving, which clearly explains how to isolate a variable. The goal is to help the student feel more comfortable with solving applied problems. Ample opportunity is provided for the student to practice translating words to symbols, which is an important part of the "Five-Step Method" of solving applied problems (discussed in modules (<link document="m21980"/>) and (<link document="m21979"/>)). Objectives of this module: be able to identify various types of equations, understand the meaning of solutions and equivalent equations, be able to solve equations of the form x + a = b and x - a = b, be familiar with and able to solve literal equations.

Overview

- Types of Equations
- Solutions and Equivalent Equations
- Literal Equations
- Solving Equations of the Form $x + a = b$ and $x - a = b$

Types of Equations

Identity
Some equations are always true. These equations are called identities. **Identities** are equations that are true for all acceptable values of the variable, that is, for all values in the domain of the equation.

$5x = 5x$ is true for all acceptable values of x.
$y + 1 = y + 1$ is true for all acceptable values of y.
$2 + 5 = 7$ is true, and no substitutions are necessary.

Contradiction
Some equations are never true. These equations are called contradictions. **Contradictions** are equations that are never true regardless of the value substituted for the variable.

$x = x + 1$ is never true for any acceptable value of x.
$0 \cdot k = 14$ is never true for any acceptable value of k.
$2 = 1$ is never true.

Conditional Equation
The truth of some equations is conditional upon the value chosen for the variable. Such equations are called conditional equations. **Conditional equations** are equations that are true for at least one replacement of the variable and false for at least one replacement of the variable.

$x + 6 = 11$ is true only on the condition that $x = 5$.
$y - 7 = -1$ is true only on the condition that $y = 6$.

Solutions and Equivalent Equations

Solutions and Solving an Equation

The collection of values that make an equation true are called **solutions** of the equation. An equation is **solved** when all its solutions have been found.

Equivalent Equations
Some equations have precisely the same collection of solutions. Such equations are called **equivalent equations**. The equations
$2x+1=7, \quad 2x=6 \quad \text{and} \quad x=3$
are equivalent equations because the only value that makes each one true is 3.

Sample Set A

Tell why each equation is an identity, a contradiction, or conditional.

Example:
The equation $x-4=6$ is a conditional equation since it will be true only on the condition that $x=10$.

Example:
The equation $x-2=x-2$ is an identity since it is true for all values of x. For example,
$$\begin{aligned} \text{if } x &= 5, & 5-2 &= 5-2 \text{ is true} \\ x &= -7, & -7-2 &= -7-2 \text{ is true} \end{aligned}$$

Example:
The equation $a+5=a+1$ is a contradiction since every value of a produces a false statement. For example,
$$\begin{aligned} \text{if } a &= 8, & 8+5 &= 8+1 \text{ is false} \\ \text{if } a &= -2, & -2+5 &= -2+1 \text{ is false} \end{aligned}$$

Practice Set A

For each of the following equations, write "identity," "contradiction," or "conditional." If you can, find the solution by making an educated guess based on your knowledge of arithmetic.
Exercise:

Problem: $x+1=10$

Solution:

conditional, $x=9$

Exercise:

Problem: $y - 4 = 7$

Solution:

conditional, $y = 11$

Exercise:

Problem: $5a = 25$

Solution:

conditional, $a = 5$

Exercise:

Problem: $\frac{x}{4} = 9$

Solution:

conditional, $x = 36$

Exercise:

Problem: $\frac{18}{b} = 6$

Solution:

conditional, $b = 3$

Exercise:

Problem: $y - 2 = y - 2$

Solution:

identity

Exercise:

Problem: $x + 4 = x - 3$

Solution:

contradiction

Exercise:

Problem: $x + x + x = 3x$

Solution:

identity

Exercise:

Problem: $8x=0$

Solution:

conditional, $x=0$

Exercise:

Problem: $m-7=-5$

Solution:

conditional, $m=2$

Literal Equations

Literal Equations
Some equations involve more than one variable. Such equations are called **literal equations**.

An equation is solved for a particular variable if that variable alone equals an expression that does not contain that particular variable.
The following equations are examples of literal equations.

1. $y=2x+7$. It is solved for y.
2. $d=rt$. It is solved for d.
3. $I=prt$. It is solved for I.
4. $z=\frac{x-u}{s}$. It is solved for z.
5. $y+1=x+4$. This equation is not solved for any particular variable since no variable is isolated.

Solving Equation of the form $x+a=b$ and $x-a=b$

Recall that the equal sign of an equation indicates that the number represented by the expression on the left side is the same as the number represented by the expression on the right side.

$$\begin{array}{ccc} \text{This} & \text{is the} & \text{this} \\ \text{number} & \text{same as} & \text{number} \\ \downarrow & \downarrow & \downarrow \\ x & = & 6 \\ x+2 & = & 8 \\ x-1 & = & 5 \end{array}$$

This suggests the following procedures:

1. We can obtain an equivalent equation (an equation having the same solutions as the original equation) by **adding** the **same number** to **both sides** of the equation.
2. We can obtain an equivalent equation by **subtracting** the **same number** from **both sides** of the equation.

We can use these results to isolate x, thus solving for x.

Example:
Solving $x+a=b$ for x

$$\begin{aligned} x+a &= b && \text{The } a \text{ is associated with } x \text{ by addition. Undo the association} \\ x+a-a &= b-a && \text{by subtracting } a \text{ from } \textit{both} \text{ sides.} \\ x+0 &= b-a && a-a=0 \text{ and } 0 \text{ is the additive identity. } x+0=x. \\ x &= b-a && \text{This equation is equivalent to the first equation, and it is} \\ & && \text{solved for } x. \end{aligned}$$

Example:
Solving $x-a=b$ for x

$$\begin{aligned} x-a &= b && \text{The } a \text{ is associated with } x \text{ by subtraction. Undo the association} \\ x-a+a &= b+a && \text{by adding } a \text{ to } \textit{both} \text{ sides.} \\ x+0 &= b+a && -a+a=0 \text{ and } 0 \text{ is the additive identity. } x+0=x. \\ x &= b+a && \text{This equation is equivalent to the first equation, and it is} \\ & && \text{solved for } x. \end{aligned}$$

Example:
Method for Solving $x+a=b$ and $x-a=b$ for x
To solve the equation $x+a=b$ for x, **subtract** a from **both** sides of the equation.
To solve the equation $x-a=b$ for x, **add** a to **both** sides of the equation.

Sample Set B

Example:
Solve $x+7=10$ for x.

$$\begin{array}{rcll} x+7 & = & 10 & \text{7 is associated with } x \text{ by addition. Undo the association} \\ x+7-7 & = & 10-7 & \text{by subtracting 7 from } \textit{both} \text{ sides.} \\ x+0 & = & 3 & 7-7=0 \text{ and 0 is the additive identity. } x+0=x. \\ x & = & 3 & x \text{ is isolated, and the equation } x=3 \text{ is equivalent to the} \\ & & & \text{original equation } x+7=10. \text{ Therefore, these two} \\ & & & \text{equation have the same solution. The solution to } x=3 \\ & & & \text{is clearly 3. Thus, the solution to } x+7=10 \text{ is also 3.} \end{array}$$

Check: Substitute 3 for x in the original equation.

$$\begin{array}{rcll} x+7 & = & 10 & \\ 3+7 & = & 10 & \text{Is this correct?} \\ 10 & = & 10 & \text{Yes, this is correct.} \end{array}$$

Example:

Solve $m-2=-9$ for m.

$$\begin{array}{rcll} m-2 & = & -9 & \text{2 is associated with } m \text{ by subtraction. Undo the association} \\ m-2+2 & = & -9+2 & \text{by adding 2 from } \textit{both} \text{ sides.} \\ m+0 & = & -7 & -2+2=0 \text{ and 0 is the additive identity. } m+0=m. \\ m & = & -7 & \end{array}$$

Check: Substitute -7 for m in the original equation.

$$\begin{array}{rcll} m-2 & = & -9 & \\ -7-2 & = & -9 & \text{Is this correct?} \\ -9 & = & -9 & \text{Yes, this is correct.} \end{array}$$

Example:

Solve $y-2.181=-16.915$ for y.

$$\begin{array}{rcl} y-2.181 & = & -16.915 \\ y-2.181+2.181 & = & -16.915+2.181 \\ y & = & -14.734 \end{array}$$

On the Calculator

Type	16.915
Press	$\boxed{+/-}$
Press	$\boxed{+}$
Type	2.181
Press	$\boxed{=}$
Display reads:	-14.734

Example:

Solve $y+m=s$ for y.

$$\begin{array}{rcll} y+m &=& s & m \text{ is associated with } y \text{ by addition. Undo the association} \\ y+m-m &=& s-m & \text{by subtracting } m \text{ from } \textit{both} \text{ sides.} \\ y+0 &=& s-m & m-m=0 \text{ and } 0 \text{ is the additive identity. } y+0=y. \\ y &=& s-m & \end{array}$$

Check: Substitute $s-m$ for y in the original equation.

$$\begin{array}{rclll} y+m &=& s & & \\ s-m+m &=& s & & \text{Is this correct?} \\ s &=& s & \text{True} & \text{Yes, this is correct.} \end{array}$$

Example:

Solve $k-3h=-8h+5$ for k.

$$\begin{array}{rcll} k-3h &=& -8h+5 & 3h \text{ is associated with } k \text{ by subtraction. Undo the association} \\ k-3h+3h &=& -8h+5+3h & \text{by adding } 3h \text{ to } \textit{both} \text{ sides.} \\ k+0 &=& -5h+5 & -3h+3h=0 \text{ and } 0 \text{ is the additive identity. } k+0=k. \\ k &=& -5h+5 & \end{array}$$

Practice Set B

Exercise:

Problem: Solve $y-3=8$ for y.

Solution:

$y=11$

Exercise:

Problem: Solve $x + 9 = -4$ for x.

Solution:

$x = -13$

Exercise:

Problem: Solve $m + 6 = 0$ for m.

Solution:

$m = -6$

Exercise:

Problem: Solve $g - 7.2 = 1.3$ for g.

Solution:

$g = 8.5$

Exercise:

Problem: solve $f + 2d = 5d$ for f.

Solution:

$f = 3d$

Exercise:

Problem: Solve $x + 8y = 2y - 1$ for x.

Solution:

$x = -6y - 1$

Exercise:

Problem: Solve $y + 4x - 1 = 5x + 8$ for y.

Solution:

$y = x + 9$

Exercises

For the following problems, classify each of the equations as an identity, contradiction, or conditional equation.

Exercise:

Problem: $m+6=15$

Solution:

conditional

Exercise:

Problem: $y-8=-12$

Exercise:

Problem: $x+1=x+1$

Solution:

identity

Exercise:

Problem: $k-2=k-3$

Exercise:

Problem: $g+g+g+g=4g$

Solution:

identity

Exercise:

Problem: $x+1=0$

For the following problems, determine which of the literal equations have been solved for a variable. Write "solved" or "not solved."

Exercise:

Problem: $y=3x+7$

Solution:

solved

Exercise:

Problem: $m = 2k + n - 1$

Exercise:

Problem: $4a = y - 6$

Solution:

not solved

Exercise:

Problem: $hk = 2k + h$

Exercise:

Problem: $2a = a + 1$

Solution:

not solved

Exercise:

Problem: $5m = 2m - 7$

Exercise:

Problem: $m = m$

Solution:

not solved

For the following problems, solve each of the conditional equations.

Exercise:

Problem: $h - 8 = 14$

Exercise:

Problem: $k + 10 = 1$

Solution:

$k = -9$

Exercise:

Problem: $m - 2 = 5$

Exercise:

Problem: $y + 6 = -11$

Solution:

$y = -17$

Exercise:

Problem: $y - 8 = -1$

Exercise:

Problem: $x + 14 = 0$

Solution:

$x = -14$

Exercise:

Problem: $m - 12 = 0$

Exercise:

Problem: $g + 164 = -123$

Solution:

$g = -287$

Exercise:

Problem: $h - 265 = -547$

Exercise:

Problem: $x + 17 = -426$

Solution:

$x = -443$

Exercise:

Problem: $h - 4.82 = -3.56$

Exercise:

Problem: $y + 17.003 = -1.056$

Solution:

$y = -18.059$

Exercise:

Problem: $k + 1.0135 = -6.0032$

Exercise:

Problem: Solve $n + m = 4$ for n.

Solution:

$n = 4 - m$

Exercise:

Problem: Solve $P + 3Q - 8 = 0$ for P.

Exercise:

Problem: Solve $a + b - 3c = d - 2f$ for b.

Solution:

$b = -a + 3c + d - 2f$

Exercise:

Problem: Solve $x - 3y + 5z + 1 = 2y - 7z + 8$ for x.

Exercise:

Problem: Solve $4a - 2b + c + 11 = 6a - 5b$ for c.

Solution:

$c = 2a - 3b - 11$

Exercises for Review

Exercise:

Problem: ([link]) Simplify $(4x^5y^2)^3$.

Exercise:

Problem: ([link]) Write $\frac{20x^3y^7}{5x^5y^3}$ so that only positive exponents appear.

Solution:

$\frac{4y^4}{x^2}$

Exercise:

Problem:

([link]) Write the number of terms that appear in the expression $5x^2 + 2x - 6 + (a + b)$, and then list them.

Exercise:

Problem: **([link])** Find the product. $(3x - 1)^2$.

Solution:

$9x^2 - 6x + 1$

Exercise:

Problem: **([link])** Specify the domain of the equation $y = \frac{5}{x-2}$.

Equations and inequalities: Solving linear equations

Strategy for Solving Equations

This chapter is all about solving different types of equations for one or two variables. In general, we want to get the unknown variable alone on the left hand side of the equation with all the constants on the right hand side of the equation. For example, in the equation $x-1=0$, we want to be able to write the equation as $x=1$.

As we saw in [review of past work]() (section on rearranging equations), an equation is like a set of weighing scales that must always be balanced. When we solve equations, we need to keep in mind that what is done to one side must be done to the other.

Method: Rearranging Equations

You can add, subtract, multiply or divide both sides of an equation by any number you want, as long as you always do it to both sides.

For example, in the equation $x+5-1=-6$, we want to get x alone on the left hand side of the equation. This means we need to subtract 5 and add 1 on the left hand side. However, because we need to keep the equation balanced, we also need to subtract 5 and add 1 on the right hand side.

Equation:

$$\begin{aligned} x+5-1 &= -6 \\ x+5-5-1+1 &= -6-5+1 \\ x+0+0 &= -11+1 \\ x &= -10 \end{aligned}$$

In another example, $\frac{2}{3}x=8$, we must divide by 2 and multiply by 3 on the left hand side in order to get x alone. However, in order to keep the equation balanced, we must also divide by 2 and multiply by 3 on the right hand side.

Equation:

$$\begin{aligned} \frac{2}{3}x &= 8 \\ \frac{2}{3}x \div 2 \times 3 &= 8 \div 2 \times 3 \\ \frac{2}{2} \times \frac{3}{3} \times x &= \frac{8 \times 3}{2} \\ 1 \times 1 \times x &= 12 \\ x &= 12 \end{aligned}$$

These are the basic rules to apply when simplifying any equation. In most cases, these rules have to be applied more than once, before we have the unknown variable on the left hand side of the equation.

Note: The following must also be kept in mind:

1. Division by 0 is undefined.
2. If $\frac{x}{y}=0$, then $x=0$ and $y \neq 0$, because division by 0 is undefined.

We are now ready to solve some equations!

Investigation : Strategy for Solving Equations

In the following, identify what is wrong.
Equation:

$$\begin{aligned} 4x-8 &= 3(x-2) \\ 4(x-2) &= 3(x-2) \\ \frac{4(x-2)}{(x-2)} &= \frac{3(x-2)}{(x-2)} \\ 4 &= 3 \end{aligned}$$

Solving Linear Equations

The simplest equation to solve is a linear equation. A linear equation is an equation where the power of the variable(letter, e.g. x) is 1(one). The following are examples of linear equations.
Equation:

$$\begin{aligned} 2x+2 &= 1 \\ \frac{2-x}{3x+1} &= 2 \\ \frac{4}{3}x-6 &= 7x+2 \end{aligned}$$

In this section, we will learn how to find the value of the variable that makes both sides of the linear equation true. For example, what value of x makes both sides of the very simple equation, $x+1=1$ true.

Since the definition of a linear equation is that if the variable has a highest power of one (1), there is at most *one solution* or *root* for the equation.

This section relies on all the methods we have already discussed: multiplying out expressions, grouping terms and factorisation. Make sure that you are comfortable with these methods, before trying out the work in the rest of this chapter.
Equation:

$$\begin{aligned} 2x+2 &= 1 \\ 2x &= 1-2 \quad \text{(like terms together)} \\ 2x &= -1 \quad \text{(simplified as much a possible)} \end{aligned}$$

Now we see that $2x=-1$. This means if we divide both sides by 2, we will get:
Equation:

$$x=-\frac{1}{2}$$

If we substitute $x=-\frac{1}{2}$, back into the original equation, we get:
Equation:

$$\begin{aligned} LHS &= 2x+2 \\ &= 2\left(-\frac{1}{2}\right)+2 \\ &= -1+2 \\ &= 1 \\ and & \\ RHS &= 1 \end{aligned}$$

That is all that there is to solving linear equations.

Note: Solving Equations

When you have found the solution to an equation, substitute the solution into the original equation, to check your answer.

Method: Solving Equations

The general steps to solve equations are:

1. Expand (Remove) all brackets.
2. "Move" all terms with the variable to the left hand side of the equation, and all constant terms (the numbers) to the right hand side of the equals sign. Bearing in mind that the sign of the terms will change from $(+)$ to $(-)$ or vice versa, as they "cross over" the equals sign.
3. Group all like terms together and simplify as much as possible.
4. Factorise if necessary.
5. Find the solution.
6. Substitute solution into **original** equation to check answer.

Khan academy video on equations - 1
[missing_resource: http://www.youtube.com/v/f15zA0PhSek&rel=0]
Exercise:
Solving Linear Equations

Problem: Solve for x: $4-x=4$

Solution:

Determine what is given and what is required

We are given $4-x=4$ and are required to solve for x.

Determine how to approach the problem

Since there are no brackets, we can start with grouping like terms and then simplifying.

Solve the problem

Equation:

$$\begin{aligned}
4 - x &= 4 \\
-x &= 4 - 4 \quad \text{(move all constant terms(numbers) to the RHS(right hand side))} \\
-x &= 0 \quad \text{(group like terms together)} \\
-x &= 0 \quad \text{(simplify grouped terms)} \\
-x &= 0 \\
\therefore \quad x &= 0
\end{aligned}$$

Check the answer Substitute solution into original equation:

Equation: $4 - 0 = 4$

Equation: $4 = 4$

Since both sides are equal, the answer is correct.

Write the final answer

The solution of $4 - x = 4$ is $x = 0$.

Exercise:
Solving Linear Equations

Problem: Solve for x: $4(2x - 9) - 4x = 4 - 6x$

Solution:

Determine what is given and what is required

We are given $4(2x - 9) - 4x = 4 - 6x$ and are required to solve for x.

Determine how to approach the problem

We start with expanding the brackets, then grouping like terms and then simplifying.

Solve the problem **Equation:**

$$\begin{aligned}
4(2x - 9) - 4x &= 4 - 6x \\
8x - 36 - 4x &= 4 - 6x \quad \text{(expand the brackets)} \\
8x - 4x + 6x &= 4 + 36 \quad \text{move all terms with x to the LHS} \\
\text{and all constant terms to the RHS of the } &= \\
(8x - 4x + 6x) &= (4 + 36) \quad \text{(group like terms together)} \\
10x &= 40 \quad \text{(simplify grouped terms)} \\
\frac{10}{10}x &= \frac{40}{10} \quad \text{(divide both sides by 10)} \\
x &= 4
\end{aligned}$$

Check the answer Substitute solution into original equation:

Equation:

$$\begin{aligned}
4(2(4) - 9) - 4(4) &= 4 - 6(4) \\
4(8 - 9) - 16 &= 4 - 24 \\
4(-1) - 16 &= -20 \\
-4 - 16 &= -20 \\
-20 &= -20
\end{aligned}$$

Since both sides are equal to -20, the answer is correct.

Write the final answer

The solution of $4(2x - 9) - 4x = 4 - 6x$ is $x = 4$.

Exercise:

Solving Linear Equations

Problem: Solve for x: $\frac{2-x}{3x+1} = 2$

Solution:

Determine what is given and what is required

We are given $\frac{2-x}{3x+1} = 2$ and are required to solve for x.

Determine how to approach the problem

Since there is a denominator of ($3x + 1$), we can start by multiplying both sides of the equation by ($3x + 1$). But because division by 0 is not permissible, there is a restriction on a value for x. ($x \neq \frac{-1}{3}$)

Solve the problem

Equation:

$$\begin{aligned} \frac{2-x}{3x+1} &= 2 \\ (2-x) &= 2(3x+1) \\ 2-x &= 6x+2 \quad \text{(remove / expand brackets)} \\ -x-6x &= 2-2 \quad \text{move all terms containing x to the LH} \end{aligned}$$

$$\text{and all constant terms(numbers) to the RHS.}$$

$$\begin{aligned} -7x &= 0 \quad \text{(simplify grouped terms)} \\ x &= 0 \div (-7) \\ therefore \quad x &= 0 \quad \text{zero divided by any number is 0} \end{aligned}$$

Check the answer

Substitute solution into original equation:

Equation:

$$\begin{aligned} \frac{2-(0)}{3(0)+1} &= 2 \\ \frac{2}{1} &= 2 \end{aligned}$$

Since both sides are equal to 2, the answer is correct.

Write the final answer

The solution of $\frac{2-x}{3x+1} = 2$ is $x = 0$.

Exercise:
Solving Linear Equations

Problem: Solve for x: $\frac{4}{3}x - 6 = 7x + 2$

Solution:

Determine what is given and what is required

We are given $\frac{4}{3}x - 6 = 7x + 2$ and are required to solve for x.

Determine how to approach the problem

We start with multiplying each of the terms in the equation by 3, then grouping like terms and then simplifying.

Solve the problem

Equation:

$$
\begin{aligned}
\frac{4}{3}x - 6 &= 7x + 2 \\
4x - 18 &= 21x + 6 \quad \text{(each term is multiplied by 3)} \\
4x - 21x &= 6 + 18 \quad \text{(move all terms with x to the LHS}
\end{aligned}
$$

$$\text{and all constant terms to the RHS of the } =)$$

$$
\begin{aligned}
-17x &= 24 \quad \text{(simplify grouped terms)} \\
\frac{-17}{-17}x &= \frac{24}{-17} \quad \text{(divide both sides by } -17) \\
x &= \frac{-24}{17}
\end{aligned}
$$

Check the answer

Substitute solution into original equation:

Equation:

$$
\begin{aligned}
\frac{4}{3} \times \frac{-24}{17} - 6 &= 7 \times \frac{-24}{17} + 2 \\
\frac{4 \times (-8)}{(17)} - 6 &= \frac{7 \times (-24)}{17} + 2 \\
\frac{(-32)}{17} - 6 &= \frac{-168}{17} + 2 \\
\frac{-32 - 102}{17} &= \frac{(-168) + 34}{17} \\
\frac{-134}{17} &= \frac{-134}{17}
\end{aligned}
$$

Since both sides are equal to $\frac{-134}{17}$, the answer is correct.

Write the final answer

The solution of $\frac{4}{3}x - 6 = 7x + 2$ is, $x = \frac{-24}{17}$.

Solving Linear Equations

1. Solve for y: $2y - 3 = 7$ [missing_resource: http://www.fhsst.org/lcR]
2. Solve for w: $-3w = 0$ [missing_resource: http://www.fhsst.org/lcR]
3. Solve for z: $4z = 16$ [missing_resource: http://www.fhsst.org/lcR]
4. Solve for t: $12t + 0 = 144$ [missing_resource: http://www.fhsst.org/lcR]
5. Solve for x: $7 + 5x = 62$ [missing_resource: http://www.fhsst.org/lcR]
6. Solve for y: $55 = 5y + \frac{3}{4}$ [missing_resource: http://www.fhsst.org/lcn]
7. Solve for z: $5z = 3z + 45$ [missing_resource: http://www.fhsst.org/lcn]
8. Solve for a: $23a - 12 = 6 + 2a$ [missing_resource: http://www.fhsst.org/lcn]
9. Solve for b: $12 - 6b + 34b = 2b - 24 - 64$ [missing_resource: http://www.fhsst.org/lcn]
10. Solve for c: $6c + 3c = 4 - 5(2c - 3)$ [missing_resource: http://www.fhsst.org/lcQ]
11. Solve for p: $18 - 2p = p + 9$ [missing_resource: http://www.fhsst.org/lcQ]
12. Solve for q: $\frac{4}{q} = \frac{16}{24}$ [missing_resource: http://www.fhsst.org/lcQ]
13. Solve for q: $\frac{4}{1} = \frac{q}{2}$ [missing_resource: http://www.fhsst.org/lcQ]
14. Solve for r: $-(-16 - r) = 13r - 1$ [missing_resource: http://www.fhsst.org/lcQ]
15. Solve for d: $6d - 2 + 2d = -2 + 4d + 8$ [missing_resource: http://www.fhsst.org/lcU]
16. Solve for f: $3f - 10 = 10$ [missing_resource: http://www.fhsst.org/lcU]
17. Solve for v: $3v + 16 = 4v - 10$ [missing_resource: http://www.fhsst.org/lcU]
18. Solve for k: $10k + 5 + 0 = -2k + -3k + 80$ [missing_resource: http://www.fhsst.org/lcU]
19. Solve for j: $8(j - 4) = 5(j - 4)$ [missing_resource: http://www.fhsst.org/lcU]
20. Solve for m: $6 = 6(m + 7) + 5m$ [missing_resource: http://www.fhsst.org/lcU]

Equations and inequalities: Linear simultaneous equations

Thus far, all equations that have been encountered have one unknown variable that must be solved for. When two unknown variables need to be solved for, two equations are required and these equations are known as simultaneous equations. The solutions to the system of simultaneous equations are the values of the unknown variables which satisfy the system of equations simultaneously, that means at the same time. In general, if there are n unknown variables, then n equations are required to obtain a solution for each of the n variables.

An example of a system of simultaneous equations is:
Equation:

$$\begin{aligned} 2x + 2y &= 1 \\ \frac{2-x}{3y+1} &= 2 \end{aligned}$$

Finding solutions

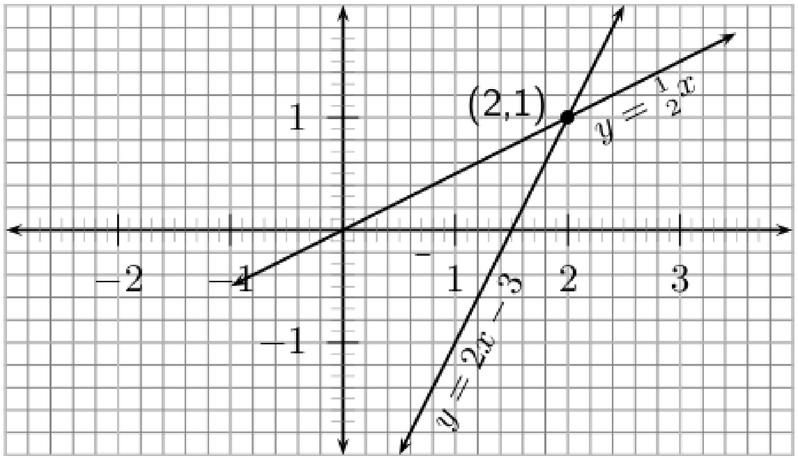
In order to find a numerical value for an unknown variable, one must have at least as many independent equations as variables. We solve simultaneous equations graphically and algebraically.
Khan academy video on simultaneous equations - 1
[missing_resource: http://www.youtube.com/v/nok99JOhcjo&rel=0]

Graphical Solution

Simultaneous equations can be solved graphically. If the graph corresponding to each equation is drawn, then the solution to the system of simultaneous equations is the co-ordinate of the point at which both graphs intersect.
Equation:

$$\begin{aligned} x &= 2y \\ y &= 2x - 3 \end{aligned}$$

Draw the graphs of the two equations in [link].



The intersection of the two graphs is $(2, 1)$. So the solution to the system of simultaneous equations in [link] is $y = 1$ and $x = 2$.

This can be shown algebraically as:

Equation:

$$\begin{array}{rrcl} & x & = & 2y \\ \text{â ˆ ´} & y & = & 2(2y) - 3 \\ & y - 4y & = & -3 \\ & -3y & = & -3 \\ & y & = & 1 \\ \text{Substitute into the first equation:} & \text{x} & = & 2(1) \\ & & = & 2 \end{array}$$

Exercise:
Simultaneous Equations

Problem: Solve the following system of simultaneous equations graphically.

Equation:

$$\begin{array}{rcl} 4y + 3x & = & 100 \\ 4y - 19x & = & 12 \end{array}$$

Solution:

Draw the graphs corresponding to each equation.

For the first equation:

Equation:

$$\begin{aligned} 4y+3x &= 100 \\ 4y &= 100-3x \\ y &= 25-\frac{3}{4}x \end{aligned}$$

and for the second equation:

Equation:

$$\begin{aligned} 4y-19x &= 12 \\ 4y &= 19x+12 \\ y &= \frac{19}{4}x+3 \end{aligned}$$

Find the intersection of the graphs.

The graphs intersect at $(4,22)$.

Write the solution of the system of simultaneous equations as given by the intersection of the graphs.

Equation:

$$\begin{aligned} x &= 4 \\ y &= 22 \end{aligned}$$

Solution by Substitution

A common algebraic technique is the substitution method: try to solve one of the equations for one of the variables and substitute the result into the other equations, thereby reducing the number of equations and the number of variables by 1. Continue until you reach a single equation with a single variable, which (hopefully) can be solved; back substitution then allows checking the values for the other variables.

In the example [link], we first solve the first equation for x:
Equation:

$$x=\frac{1}{2}-y$$

and substitute this result into the second equation:
Equation:

$$
\begin{array}{rcl}
\frac{2-x}{3y+1} & = & 2 \\
\frac{2-\left(\frac{1}{2}-y\right)}{3y+1} & = & 2 \\
2-\left(\frac{1}{2}-y\right) & = & 2(3y+1) \\
2-\frac{1}{2}+y & = & 6y+2 \\
y-6y & = & -2+\frac{1}{2}+2 \\
-5y & = & \frac{1}{2} \\
y & = & -\frac{1}{10}
\end{array}
$$

Equation:

$$
\begin{array}{rrcl}
\text{â ˆ ´} & x & = & \frac{1}{2}-y \\
 & & = & \frac{1}{2}-\left(-\frac{1}{10}\right) \\
 & & = & \frac{6}{10} \\
 & & = & \frac{3}{5}
\end{array}
$$

The solution for the system of simultaneous equations [link] is:

Equation:

$$
\begin{array}{rcl}
x & = & \frac{3}{5} \\
y & = & -\frac{1}{10}
\end{array}
$$

Exercise:

Simultaneous Equations

Problem: Solve the following system of simultaneous equations:

Equation:

$$\begin{aligned} 4y + 3x &= 100 \\ 4y - 19x &= 12 \end{aligned}$$

Solution:

Decide how to solve the problem

If the question does not explicitly ask for a graphical solution, then the system of equations should be solved algebraically.

Make the subject of the first equation.

Equation:

$$\begin{aligned} 4y + 3x &= 100 \\ 3x &= 100 - 4y \\ x &= \frac{100 - 4y}{3} \end{aligned}$$

Substitute the value obtained for into the second equation.

Equation:

$$\begin{aligned} 4y - 19\left(\frac{100 - 4y}{3}\right) &= 12 \\ 12y - 19(100 - 4y) &= 36 \\ 12y - 1900 + 76y &= 36 \\ 88y &= 1936 \\ y &= 22 \end{aligned}$$

Substitute into the equation for .

Equation:

$$\begin{aligned} x &= \frac{100 - 4(22)}{3} \\ &= \frac{100 - 88}{3} \\ &= \frac{12}{3} \\ &= 4 \end{aligned}$$

Substitute the values for and into both equations to check the solution.

Equation:

$$\begin{aligned} 4(22)+3(4)=88+12 &= 100 \\ 4(22)-19(4)=88-76 &= 12 \end{aligned}$$

Exercise:
Bicycles and Tricycles

Problem:

A shop sells bicycles and tricycles. In total there are 7 cycles (cycles includes both bicycles and tricycles) and 19 wheels. Determine how many of each there are, if a bicycle has two wheels and a tricycle has three wheels.

Solution:

Identify what is required

The number of bicycles and the number of tricycles are required.

Set up the necessary equations

If b is the number of bicycles and t is the number of tricycles, then:

Equation:

$$\begin{aligned} b+t &= 7 \\ 2b+3t &= 19 \end{aligned}$$

Solve the system of simultaneous equations using substitution.

Equation:

$$\begin{aligned} b &= 7-t \\ \text{Into second equation:}\quad 2(7-\mathrm{t})+3\mathrm{t} &= 19 \\ 14-2t+3t &= 19 \\ t &= 5 \\ \text{Into first equation: :}\quad \mathrm{b} &= 7-5 \\ &= 2 \end{aligned}$$

Check solution by substituting into original system of equations.

Equation:

$$\begin{aligned} 2+5 &= 7 \\ 2(2)+3(5)=4+15 &= 19 \end{aligned}$$

1. Solve graphically and confirm your answer algebraically: $3a - 2b7 = 0$, $a - 4b + 1 = 0$ Click here for the solution
2. Solve algebraically: $15c + 11d - 132 = 0$, $2c + 3d - 59 = 0$ Click here for the solution
3. Solve algebraically: $-18e - 18 + 3f = 0$, $e - 4f + 47 = 0$ Click here for the solution
4. Solve graphically: $x + 2y = 7$, $x + y = 0$ Click here for the solution